## A practical a posteriori strategy to determine the optimal number of DoFs for hp-refinement in finite element methods

* Introduction

In finite element methods, h-refinement, p-refinement, or a combination of them are widely used to improve the accuracy of the results. All these strategies aim at increasing the number of degrees of freedom (dofs) to decrease the truncation error.

However, with increasing number of dofs, the round-off error increases. And when the number of dofs is very large, the round-off error would exceed the truncation error, and any further refinements would result in less accurate solutions.

To find the attainable highest accuracy, the dependencies of the truncation error and round-off error on the number of degrees of freedom need to be clarified.

* results

Focusing on the solution, first-order and second-order derivative of the one-dimensional differential equations with a variety of boundary condition, we investigate both the standard and mixed FEMs using the global mesh refinement. We find that the truncation error of these variables tends to decrease along a straight line with increasing number of dofs in the log-log plot. The slope of this line for the direct solution is one order higher than the element degree. With one time of differentiation for the direct solution, the slope would increase one.

Furthermore, the round-off error for the three variables would increase along a straight line with increasing number of dofs in the log-log plot. The slope of the line is 2 for the standard FEM and 1 for the mixed FEM.

Using the double working precision, and when the L2 norm of the solution is 1 in the standard FEM, and additionally the first-order derivative is scaled by the L2 norm of the solution, the offset of this line for the three variables is about 10^{-16}.

Combining the lines of the two kinds of errors in the log-log plot, the optimal number of dofs is obtained at the intersection, which are different for each variable. To obtain the highest accuracy for all the variables, results of different refinement need to be combined.

* Improving accuracy

Furthermore, to improve the accuracy, higher-order polynomials can be used since the truncation error decreases faster by increasing the number of dofs; and the mixed FEM is recommended because the round-off error is smaller than that of the standard FEM.